GCSE
MATHEMATICS
Higher tier

Algebra

Topic test – Quadratic nth term

Mark Scheme

8300

Version 1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
3.14 …	Allow answers which begin 3.14 eg 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1(a)	38 and 51	B2	B1 for one correct.
1(b)	First and second differences correct	M1	3 5 7 9 2 2 2
	Correctly subtracts their n^2 from the given sequence	M1	2 2 2 2
	$n^2 + 2$	A1	
1(c)	$n^2 + 2 = 402$	M1	Soi $n^2 = 400$ $n = 20$
	Yes, stated with a suitable reason	A1	Reasons could include "because 402 is the 20th term" "n is a whole number".

Q	Answer	Mark	Comments
2(a)	113 and 159	B2	B1 one correct term
2(b)	**Alternative method 1**		
	First and second differences correct	M1	14 22 30 8 8
	Correctly subtracts $\dfrac{\text{their } 8}{2} n^2$ from the given sequence	M1	5 7 9 (11 13 15)
	$2n$	M1dep	
	$4n^2 + 2n + 3$	A1	oe
	Alternative method 2		
	Any three of $a + b + c = 9$ $4a + 2b + c = 23$ $9a + 3b + c = 45$ $16a + 4b + c = 75$	M1	Allow one error but all equations must be in a, b and c
	Eliminates one variable to obtain a pair of equations in two variables eg $3a + b = 14$ and $5a + b = 22$	M1	
	Eliminates one variable correctly eg $a = 4$	M1dep	Allow one error
	$4n^2 + 2n + 3$	A1	oe

Q	Answer	Mark	Comments
2(b) cont	**Alternative method 3** Sets up table of differences 	n	0	1	2	3	4
c	3	9	23	45	75
$a + b$		6	14	22	30	
$2a$			8	8	8			M1	
	Extends table back to $n = 0$	M1	
	Identifies rows as $2a$, $a + b$ and c	A1	
	$4n^2 + 2n + 3$	A1	oe

Q	Answer	Mark	Comments
3	Substitution of $n = 1, 2, 3$ or 4 seen or implied	M1	$2(1)^2 - 5(1) + 1 \ (= -2)$ $2(2)^2 - 5(2) + 2 \ (= -1)$ $2(3)^2 - 5(3) + 3 \ (= 4)$ $2(4)^2 - 5(4) + 4 \ (= 13)$
	$-2, -1, 4, 13$	A2	A1 for 2 correct.

Q	Answer	Mark	Comments
4(a)	No **with** reason eg it is a linear sequence	B1	oe The nth term is $3n - 1$ It goes up by the same amount each time, quadratic sequences don't do this.
4(b)	Yes **with** reason eg the second differences are the same	B1	oe The second difference is 2. The nth term is $n^2 + 2n - 3$
4(c)	No **with** reason It is a Fibonacci sequence		oe The second differences are not the same. The sequence is made up of adding the previous two terms.

Q	Answer	Mark	Comments
5	Second differences 6 Implied by $3n^2$	M1	
	Any three values from –2, 0, 2, 4	M1dep	
	$3n^2 + 2n - 4$	A1	oe Allow $a = 3$, $b = 2$, $c = -4$

Q	Answer	Mark	Comments
6	**Alternative method 1**		
	Cuboid 1 : $1 \times 2 \times 2$	M1	
	Cuboid 2 : $2 \times 2 \times 3$	M1	
	Cuboid 3 : $3 \times 2 \times 4$ **and** continues for at least 2 more products seen.	M1	
	Cuboid 7 : $7 \times 2 \times 8$	A1	
	Finds a formula and substitutes $n = 7$ and makes a valid conclusion eg no and $112 > 100$.	A1	NB SC2 112 and valid conclusion with no working.
	Alternative method 2		
	First four cuboids have 4, 12, 24, 40 cubes	M1	
	Recognises the rule $+8$, $+12$, $+16$ and shows $+20$ or 60	M1	
	Continues the list to the 7th cuboid, showing values with at most one error 60, 84, 112	M1	
	112 for 7th value	A1	
	Makes a valid conclusion based on their 7th term, eg no and $112 > 100$.	A1	

Q	Answer	Mark	Comments
	Alternative method 3		

6 (cont)	4 12 24 40
	
	    8 12 16
	
	     4 4
	
	Numbers of cubes identified and second difference calculated
		M1	

	$2n^2$	M1	
	2 4 6 $(+2n)$ Difference between $2n^2$ and original series calculated	M1	
	$2n^2 + 2n$ $2(7)^2 + 2 \times 7$	A1	
	Finds a quadratic formula starting with $2n^2$ and substitutes $n = 7$ and makes a valid conclusion eg no and 112 (> 100)	A1	