GCSE

MATHEMATICS

Higher tier

Geometry and measures

Topic test – Vectors

Mark Scheme

8300

Version 1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
3.14 …	Allow answers which begin 3.14 eg 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1	$\begin{pmatrix}1\\-2\end{pmatrix}$	B2	B1 for 1 correct value in correct position or unsimplified equivalent single vector eg $\begin{pmatrix}-2--3\\3-5\end{pmatrix}$
	Additional Guidance		
	Condone fraction line for B2 or B1, eg $\begin{pmatrix}\frac{1}{-2}\end{pmatrix}$		B2
2(a)	$\begin{pmatrix}12\\-11\end{pmatrix}$	B2	B1 for 1 correct value in correct position or unsimplified equivalent single vector eg $\begin{pmatrix}2+6+4\\-10+-2+1\end{pmatrix}$
	Additional Guidance		
	Condone fraction line for B2 or B1, eg $\begin{pmatrix}\frac{12}{-11}\end{pmatrix}$		B2
2(b)	$\begin{pmatrix}18\\-6\end{pmatrix}$ seen	M1	condone missing bracket and/or divisor line
	Valid reason	A1	eg $\begin{pmatrix}18\\-6\end{pmatrix} = 3 \times \begin{pmatrix}6\\-2\end{pmatrix}$ or $\begin{pmatrix}18\\-6\end{pmatrix} = 3\mathbf{b}$ or $\begin{pmatrix}18\\-6\end{pmatrix}$ is a multiple of $\begin{pmatrix}6\\-2\end{pmatrix}$ or $\mathbf{a} + 4\mathbf{c}$ is a multiple of $\mathbf{b}$ or $3\mathbf{b} = \mathbf{a} + 4\mathbf{c}$

Q	Answer	Mark	Comments
3	$\begin{pmatrix}5\\-6\end{pmatrix}$	B2	B1 for 1 correct value in correct position or unsimplified equivalent single vector, eg $\begin{pmatrix}3+2\\-1+-5\end{pmatrix}$
	Additional Guidance		
	Condone fraction line for B2 or B1, eg $\begin{pmatrix}\frac{5}{-6}\end{pmatrix}$		B2
4(a)	$-\frac{1}{2}\mathbf{q}$	B1	oe
4(b)	$\begin{pmatrix}8\\-2\end{pmatrix}$ drawn on the grid with direction shown	B2	$\pm\frac{1}{4}$ centimetre square B1 for $\begin{pmatrix}8\\-2\end{pmatrix}$ seen in working or correct line drawn with incorrect direction or no direction shown or correctly joined vectors for **c** and $-\mathbf{d}$ with correct directions shown
	Additional Guidance		
	Mark intention. Line does not need to be ruled and ignore all labelling for **c**, **d**, and **c** – **d**		
5(a)	$-(3\mathbf{a}+\mathbf{b})+5\mathbf{a}-2\mathbf{b}$ or $-3\mathbf{a}-\mathbf{b}+5\mathbf{a}-2\mathbf{b}$	M1	may be seen on diagram
	$2\mathbf{a}-3\mathbf{b}$	A1	
5(b)	$8\mathbf{a}-12\mathbf{b}$	B1ft	their **(a)** × 4

Q	Answer	Mark	Comments
6	–8**a** – 2**b** + 5**a** + 6**b** or –3**a** + 4**b**	M1	$\overrightarrow{EH}$ may be seen on diagram or as part of a longer vector sum
	2 × their –3**a** + 4**b** or –6**a** + 8**b**	M1dep	$\overrightarrow{FG}$ may be seen on diagram
	Any correct path from *F* to *E* eg their (–6**a** + 8**b**) + (**a** – 9**b**) eg their (–6**a** + 8**b**) – (–**a** + 9**b**) eg their (–6**a** + 8**b**) + their (–2**a** – 5**b**) + their (–5**a** – 6**b**) + their (8**a** + 2**b**)	M1dep	$\overrightarrow{FG} + \overrightarrow{GE}$ $\overrightarrow{FG} - \overrightarrow{EG}$ $\overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{DE}$
	–5**a** – **b**	A1	SC3 5**a** + **b** or **b** + 5**a**
	Additional Guidance		
	Missing brackets and/or incorrect additions or subtractions can be recovered for all four marks eg 6**a** + 8**b** + **a** – 9**b** recovered to –5**a** – **b**		M3A1
	Marks can only be awarded for expressions in terms of **a** and **b**		

7	$-\frac{1}{5}$**a**	B1	oe

8	$(\overrightarrow{NQ} =) \frac{5}{2} \times 4$**b** or 10**b**	M1	oe eg $(\overrightarrow{QN} =)$ –10**b**
	$(\overrightarrow{QR} =)$ – (their 10**b** + 4**b**) – 9**b** or –14**b** – 9**a**	M1dep	
	$\frac{1}{2}$(–14**b** – 9**a**) or –7**b** – $\frac{9}{2}$**a**	A1	oe eg –7**b** – 4.5**a** SC2 7**b** + $\frac{9}{2}$**a** **or** 7**b** + 4.5**a**

Q	Answer	Mark	Comments
9	($\overrightarrow{CD}$ =) 2**a** or ($\overrightarrow{AC}$ =) 3**b** – 2**a**	M1	may be on diagram or as part of a longer vector sum oe eg ($\overrightarrow{CA}$ =) –3**b** + 2**a**
	($\overrightarrow{AF}$ =) $\frac{2}{3}$(their 3**b** – 2**a**) or 2**b** – $\frac{4}{3}$**a** or ($\overrightarrow{FC}$ =) $\frac{1}{3}$(3**b** – 2**a**) or **b** – $\frac{2}{3}$**a**	M1dep	oe eg 2**b** – 1.33(…)**a**
	Any correct path from *E* to *F* eg **a** – 3**b** + 2**b** – $\frac{4}{3}$**a** eg **a** – 3**b** – 2**a** + 3**b** – $\frac{1}{3}$(their 3**b** – 2**a**)	M1dep	$\overrightarrow{ED}+\overrightarrow{DA}+\overrightarrow{AF}$ $\overrightarrow{ED}+\overrightarrow{DA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CF}$
	$-\frac{1}{3}$**a** – **b**	A1	oe eg –1.33(…)**a** – **b**

Q	Answer	Mark	Comments
10	($\overrightarrow{AC}$ =) 2**a** + 3**b**	M1	oe may be on diagram
	($\overrightarrow{NM}$ =) **a** + $\frac{3}{2}$**b**	M1	oe may be on diagram
	Valid reason	A1	eg 2**a** + 3**b** = 2 × (**a** + $\frac{3}{2}$**b**) or 2**a** + 3**b** is a multiple of **a** + $\frac{3}{2}$**b** or $\overrightarrow{AC}$ is a multiple of $\overrightarrow{NM}$ or $\overrightarrow{AC}$ = $2\overrightarrow{NM}$

Q	Answer	Mark	Comments
11(a)	$\overrightarrow{BQ} = \frac{3}{4} \times 8\mathbf{b}$ or $6\mathbf{b}$ **or** $\overrightarrow{QC} = \frac{1}{4} \times 8\mathbf{b}$ or $2\mathbf{b}$ **or** $\overrightarrow{AC} = 8\mathbf{b} - 6\mathbf{a}$ or $\overrightarrow{AP}$ or $\overrightarrow{PC} = 4\mathbf{b} - 3\mathbf{a}$	M1	oe eg $\overrightarrow{QB} = -6\mathbf{b}$ may be seen on diagram or as part of a larger vector sum oe eg $\overrightarrow{CQ} = -2\mathbf{b}$ oe eg $\overrightarrow{CA} = 6\mathbf{a} - 8\mathbf{b}$ oe eg $\overrightarrow{PA} = 3\mathbf{a} - 4\mathbf{b}$
	Any correct path from *P* to *Q* eg $(\overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BQ} =)$ $3\mathbf{a} - 4\mathbf{b} - 6\mathbf{a} + 6\mathbf{b}$ or $(\overrightarrow{PC} + \overrightarrow{CQ} =)$ $4\mathbf{b} - 3\mathbf{a} - 2\mathbf{b}$	M1dep	
	$2\mathbf{b} - 3\mathbf{a}$	A1	
11(b)	Any valid reason	B1	eg $\overrightarrow{AB}$ is not a multiple of $\overrightarrow{PQ}$ or $\overrightarrow{2PQ} = 4\mathbf{b} - 6\mathbf{a}$ and does not equal $-6\mathbf{a}$

Q	Answer	Mark	Comments
12	$(\overrightarrow{OP}) = 12\mathbf{b} - k\mathbf{b}$ or $(\overrightarrow{OQ}) = 5\mathbf{a}$ or $(\overrightarrow{QC}) = 10\mathbf{a}$ or $(\overrightarrow{BC}) = 15\mathbf{a} - 12\mathbf{b}$	M1	oe may be on diagram or as part of a larger vector sum
	$(\overrightarrow{AP}) = 5\mathbf{a} - 4\mathbf{b} - k\mathbf{b}$ or $5\mathbf{a} - \mathbf{b}(4 + k)$ or $(\overrightarrow{PQ}) = -(12\mathbf{b} - k\mathbf{b}) + 5\mathbf{a}$ or $\mathbf{b}(-12 + k) - 5\mathbf{a}$ or $(\overrightarrow{AQ}) = 5\mathbf{a} - 4\mathbf{b} - 12\mathbf{b} + 5\mathbf{a}$ or $10\mathbf{a} - 16\mathbf{b}$	M1dep	oe eg $(\overrightarrow{AP}) = 20\mathbf{a} - 16\mathbf{b} - 15\mathbf{a} + 12\mathbf{b} - k\mathbf{b}$ oe eg $(\overrightarrow{PQ}) = k\mathbf{b} + 15\mathbf{a} - 12\mathbf{b} - 10\mathbf{a}$ oe eg $(\overrightarrow{AQ}) = 20\mathbf{a} - 16\mathbf{b} - 10\mathbf{a}$
	$(\overrightarrow{AP}) = 5\mathbf{a} - 4\mathbf{b} - k\mathbf{b}$ or $5\mathbf{a} - \mathbf{b}(4 + k)$ and $(\overrightarrow{AQ}) = 5\mathbf{a} - 4\mathbf{b} - 12\mathbf{b} + 5\mathbf{a}$ or $10\mathbf{a} - 16\mathbf{b}$ or $(\overrightarrow{PQ}) = -(12\mathbf{b} - k\mathbf{b}) + 5\mathbf{a}$ or $\mathbf{b}(-12 + k) - 5\mathbf{a}$ and $(\overrightarrow{AQ}) = 5\mathbf{a} - 4\mathbf{b} - 12\mathbf{b} + 5\mathbf{a}$ or $10\mathbf{a} - 16\mathbf{b}$	M1dep	oe eg $(\overrightarrow{AP}) = 20\mathbf{a} - 16\mathbf{b} - 15\mathbf{a} + 12\mathbf{b} - k\mathbf{b}$ oe eg $(\overrightarrow{AQ}) = 20\mathbf{a} - 16\mathbf{b} - 10\mathbf{a}$ oe eg $(\overrightarrow{PQ}) = k\mathbf{b} + 15\mathbf{a} - 12\mathbf{b} - 10\mathbf{a}$ oe eg $(\overrightarrow{AQ}) = 20\mathbf{a} - 16\mathbf{b} - 10\mathbf{a}$
	$(\overrightarrow{AQ} = -\overrightarrow{2PQ})$ $-2\mathbf{b}(-12 + k) = 16\mathbf{b}$ or $-12 + k = -8$ or $(\overrightarrow{AQ} = \overrightarrow{2AP})$ $2\mathbf{b}(4 + k) = 16\mathbf{b}$ or $4 + k = 8$	M1dep	setting up an equation in terms of k oe eg $10\mathbf{a} - 16\mathbf{b} = -2(-12\mathbf{b} + k\mathbf{b} - 5\mathbf{a})$ oe eg $10\mathbf{a} - 16\mathbf{b} = 2(5\mathbf{a} - 4\mathbf{b} - k\mathbf{b})$
	$(k =)\ 4$	A1	