Solve equations by iteration

H

1 **a)** A sequence is given by the rule $u_{n+1} = u_n + 3$

Given that $u_1 = 10$, work out the values of u_2, u_3 and u_4

$u_2 =$ ☐ $u_3 =$ ☐ $u_4 =$ ☐

b) Another sequence is given by the rule $a_{n+1} = 3a_n$.

Given that $a_1 = 8$, work out the values of a_2, a_3 and a_4

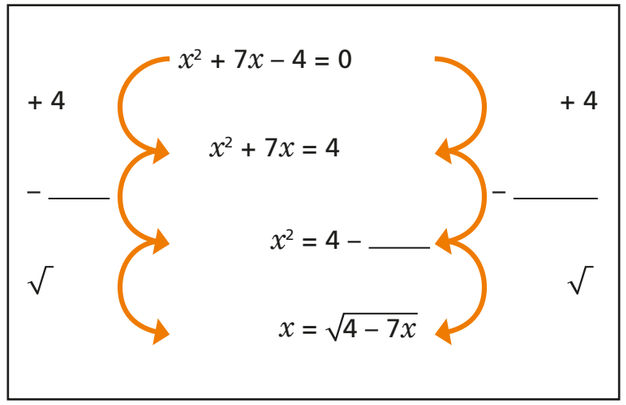
$a_2 =$ ☐ $a_3 =$ ☐ $a_4 =$ ☐

c) A third sequence is given by the rule $b_{n+1} = 10 - 2b_n$.

Given that $b_1 = 4$, work out the values of b_2, b_3 and b_4

$b_2 =$ ☐ $b_3 =$ ☐ $b_4 =$ ☐

2 Complete the workings to show that $x^2 + 7x - 4 = 0$ can be rearranged to $x = \sqrt{4 - 7x}$.

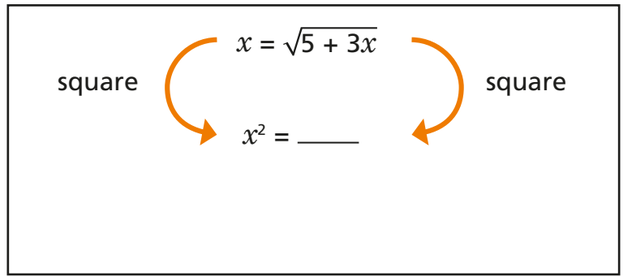


3 $x_{n+1} = \sqrt{5 + 3x_n}$

a) Given that $x_0 = 3$, work out the values of x_1, x_2, x_3 and x_4 to 3 significant figures.

$x_1 =$ ☐ $x_2 =$ ☐ $x_3 =$ ☐ $x_4 =$ ☐

b) Complete the workings to show that $x = \sqrt{5 + 3x}$ can be rearranged to $x^2 - 3x - 5 = 0$



c) Use the quadratic formula to find the roots of $x^2 - 3x - 5 = 0$ correct to 2 decimal places.

first root = ☐ second root = ☐

4 **a)** Show that the equation $x^3 + 5x - 1 = 0$ has a root between 0 and 1

b) Show that the equation $x^3 + 5x - 1 = 0$ can be rearranged to $x = \frac{1}{5}(1 - x^3)$.

c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{5}(1 - x_n^3)$ to find an estimate of the root of the equation $x^3 + 5x - 1 = 0$

$x =$ ☐

5 **a)** Show that the equation $x^3 - 7x = 3$ has a root between –1 and 0

b) Show that the equation $x^3 - 7x = 3$ can be rearranged to $x = \frac{x^3 - 3}{7}$

c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{x_n^3 - 3}{7}$ to find an estimate of a root of the equation $x^3 - 7x = 3$

$x =$ ☐

Investigate the iteration for other values of x_0, for example $x_0 = 1$, $x_0 = 2$, $x_0 = -1$, etc.

d) Show that the equation $x^3 - 7x = 3$ can also be rearranged to $x = \sqrt[3]{3 + 7x}$

e) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \sqrt[3]{3 + 7x_n}$ to find an estimate of a root of the equation $x^3 - 7x = 3$

$x =$ ☐

Investigate the iteration for other values of x_0, for example $x_0 = 1$, $x_0 = 2$, $x_0 = -1$, etc.

6

$$x_{n+1} = \frac{2}{x_n^2} - 4$$

a) Use $x_0 = -2$ to find the values of x_1, x_2, x_3 and x_4

$x_1 =$ ☐ $x_2 =$ ☐ $x_3 =$ ☐ $x_4 =$ ☐

b) Explain the relationship between x_1, x_2, x_3 and x_4 and the equation $x^3 + 4x^2 - 2 = 0$